

Virtual Gravitons at the LHC

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Abstract. Virtual gravitons effects at the LHC in scenarios with large extra dimensions and low-scale gravity are sensitive to the ultraviolet completion of the Kaluza-Klein effective theory. We study implications of a gravitational fixed point at high energies on gravitational Drell-Yan lepton production in hadron collisions. The fixed point behaviour leads to finite LHC cross sections. We determine the reach for the fundamental Planck scale. An observation of these signals might shed light on the fundamental quantum theory of gravity.

PACS. 04.60.-m Quantum gravity – 04.50.+h Gravity in more than four dimensions – 11.10.Hi Renormalization group evolution of parameters – 11.15.Tk Other nonperturbative techniques

1 Introduction

Theories with large compactified extra dimensions [1] have been studied in detail for different colliders [2]. In such models gravity propagates in the higher-dimensional bulk, while Standard Model particles are typically confined to the four-dimensional brane. If the fundamental Planck scale M_D in $(4+n)$ dimensions is indeed in the TeV range, the LHC will be likely to see clear signals thereof. This way the LHC becomes sensitive to the dynamics of gravity, and could possibly become the first experiment able to establish evidence for the quantization of gravity. Searches for massive Kaluza-Klein (KK) gravitons at hadron colliders are based on two signatures: real graviton emission, leading to missing transverse momentum [2, 3] and virtual graviton effects which alter the rates and distributions of Standard Model candles like Drell-Yan or photon-pair production [2, 4]. In the context of low-scale quantum gravity, such signals have been studied in a KK effective field theory [2], which allows for a controlled description as long as the relevant momentum scales are sufficiently below an ultraviolet (UV) cutoff of the order of the fundamental Planck scale M_D .

For momentum transfer near the Planck scale and above, an understanding of gravitational interactions requires an explicit quantum theory for gravity. It has been suggested that a local quantum theory of gravity in terms of the metric field may very well exist on a non-perturbative level, despite its notorious perturbative non-renormalizability [5]. This “asymptotic safety” scenario requires the existence of a non-trivial UV fixed point for quantum gravity under the renormalization group. In higher dimensions, as relevant for the present setup, a new non-trivial UV fixed point has been detected in [6, 7]. The fixed point implies that gravitational interactions become soft at high ener-

gies. These findings also support $4d$ results based on renormalization group studies in various setups and approximations [8, 9, 10, 11, 12, 13, 14, 6, 15, 16], and lattice simulations [17]. With [6, 7] at hand, it is now feasible to evaluate phenomenological implications for quantum gravity at the LHC [7, 18, 19, 20].

Real graviton emission is weakly sensitive to the UV sector of the theory, and an effective-field-theory approach already yields stable cross section predictions at the LHC. This is further helped through the steep drop in the gluon densities, which acts as an additional theory-independent UV cutoff. Virtual graviton effects, in turn, induce Planck-scale suppressed higher-dimensional operators which are dominated by the far UV regime of the KK spectrum. Within effective theory, this requires an UV regularisation. Furthermore, this strong cutoff sensitivity implies large theoretical uncertainties on production rates at the LHC and beyond [25, 26]. Here, we study the impact of a gravitational fixed-point at high energies on virtual gravitons and lepton pair production $pp \rightarrow \ell^+ \ell^-$ at the LHC using Wilson’s renormalization group [18].

2 Gravitational fixed point

We first discuss implications of gravitational fixed points and consider the renormalization group equation for the gravitational coupling G as a function of the momentum scale μ in D dimensions [6, 16]. We have

$$\beta_g \equiv \frac{d g(\mu)}{d \ln \mu} = (D - 2 + \eta) g(\mu), \quad (1)$$

where $g(\mu) = G(\mu)\mu^{D-2} \equiv G_0 Z^{-1}(\mu)\mu^{D-2}$ is the dimensionless gravitational coupling. Here $\eta = -\mu \partial_\mu \ln Z$ denotes the anomalous dimension of the graviton. The

wave function factor is normalized to $Z(\mu_0) = 1$ at some reference scale μ_0 with $G(\mu_0)$ given by Newton's constant G_0 . In general, the anomalous dimension depends on all couplings of the theory. Due to its structure, eq.(1) predicts two types of fixed points. At small coupling, the anomalous dimension vanishes and $g = 0$ corresponds to the non-interacting (*i.e.* Gaussian) fixed point. This fixed point dominates the deep infrared region of gravity $\mu \rightarrow 0$. An interacting fixed point g_* can occur if the anomalous dimension of the graviton becomes non-perturbatively large,

$$\eta_* = 2 - D. \quad (2)$$

Hence, a non-trivial fixed point of quantum gravity in $D > 2$ implies a negative value for the graviton anomalous dimension, precisely counter-balancing the canonical dimension of G . This means the gravitational coupling constant scales as $G(\mu) \rightarrow g_*/\mu^{D-2}$ in the vicinity of the non-trivial fixed point. In the UV limit the gravitational coupling $G(\mu \rightarrow \infty)$ then becomes arbitrarily weak.

For the explicit renormalization group equations for gravity [8] we consider an effective action Γ_k with

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^D x \sqrt{g} [-R(g) + \dots] \quad (3)$$

where k denotes the Wilsonian renormalization-group scale replacing the scale μ introduced in eq.(1), and $R(g)$ denotes the Ricci scalar. The dots stand for the cosmological constant, higher dimensional operators in the metric field, gravity-matter interactions, a classical gauge fixing and ghost terms. All couplings in eq.(3) become running couplings as functions of the momentum scale k . In the infrared $k \ll M_D$, the gravitational sector is well-approximated by the Einstein-Hilbert action with $G_k \approx G_0$. The corresponding operators scale canonically. In the UV regime $k \gg M_D$, the non-trivial renormalization-group running of gravitational couplings becomes important. The Wilsonian renormalization-group flow for the action eq.(3) is given by an exact differential equation [21, 8, 22, 23, 24]

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \quad (4)$$

and $t = \ln k$. The trace stands for a momentum integration and a sum over indices and fields, and $R_k(q^2)$ denotes an appropriate infrared cutoff function at momentum scale $q^2 \approx k^2$ [23].

To illustrate the leading renormalization-group effects of gravity in models with large extra dimensions and Standard-Model matter on a brane we approximate eq.(3) by the Ricci scalar and discuss the running of g_k [6]. The central pattern is not altered through the inclusion of a cosmological constant [7]. Using eq.(3) and eq.(4), we find

$$\beta_g = \frac{(1 - 4Dg)(D - 2)}{1 - (2D - 4)g} g \quad (5)$$

where g has been rescaled by a numerical factor. Eq.(5) displays a non-Gaussian fixed point at $g_* = 1/(4D)$. Integrating eq.(5), we find

$$\frac{1}{D-2} \ln \left(\frac{g_k}{g_0} \right) - \frac{1}{\theta_{\text{NG}}} \ln \left(\frac{g_* - g_k}{g_* - g_0} \right) = \ln \frac{k}{k_0} \quad (6)$$

with initial condition g_0 at $k = k_0$, and $\theta_{\text{NG}} = 2D(D-2)/(D+2)$. The result eq.(6) holds for generic Wilsonian momentum cutoff, with the slight modification that the values for g_* and the scaling exponent θ_{NG} can depend on the details [6, 7]. The anomalous dimension of the graviton reads

$$\eta = \frac{2(D-2)(D+2)g}{2(D-2)g-1}. \quad (7)$$

Inserting the running coupling eq.(6) into eq.(5) shows that the anomalous dimension displays a smooth cross-over between the IR domain $k \ll M_D$ where $\eta \approx 0$ and the UV domain $k \gg M_D$ where $\eta \approx 2 - D$. The cross-over regime becomes narrower with increasing dimension [7, 18].

3 Drell-Yan with Gravitons

Virtual graviton effects, as opposed to real graviton emission, crucially depend on an UV completion, like the UV fixed point [2]. Contributions to the Drell-Yan process can be generated through a dimension-8 operator in the effective action [2, 25, 26]. Tree-level graviton exchange is described by an amplitude $\mathcal{A} = \mathcal{S} \cdot \mathcal{T}$, where $\mathcal{T} = T_{\mu\nu} T^{\mu\nu} - T_\mu^\mu T_\nu^\nu / (2+n)$ is a function of the energy-momentum tensor, and

$$\mathcal{S} = \frac{S_{n-1}}{M_D^{2+n}} \int_0^\infty dm m^{n-1} P(s, m) \quad (8)$$

with $S_{n-1} = 2\pi^{n/2}/\Gamma(n/2)$ is a function of the scalar part $P(s, m)$ of the graviton propagator [2, 25, 26]. The integration over the KK tower m corresponds to gravity propagating in the higher-dimensional bulk. If the graviton anomalous dimension is small, the propagator is well approximated by the usual scalar graviton propagator as long as the relevant momentum transfer as well as the KK masses are below the Planck scale M_D . It is well known that this expression for \mathcal{S} is UV divergent for $n \geq 2$ [2]. Implementing an UV cutoff Λ [25] as the upper integration boundary in the KK integration over m gives

$$\mathcal{S}_\Lambda = \frac{S_{n-1}}{n-2} \frac{1}{M_D^4} \left(\frac{\Lambda}{M_D} \right)^{n-2} \left[1 + \mathcal{O} \left(\frac{s}{\Lambda^2} \right) \right]. \quad (9)$$

The strong cutoff dependence of \mathcal{S}_Λ indicates that the effective-field-theory prediction for \mathcal{S} for $n \geq 2$ is indeed dominated by UV contributions and sensitive to the UV completion of the KK theory. Note that in this approach we apply the same cutoff to the partonic LHC energy, which means we only evaluate contributions to \mathcal{S}_Λ with both, $\sqrt{s} < \Lambda$ and $m < \Lambda$.

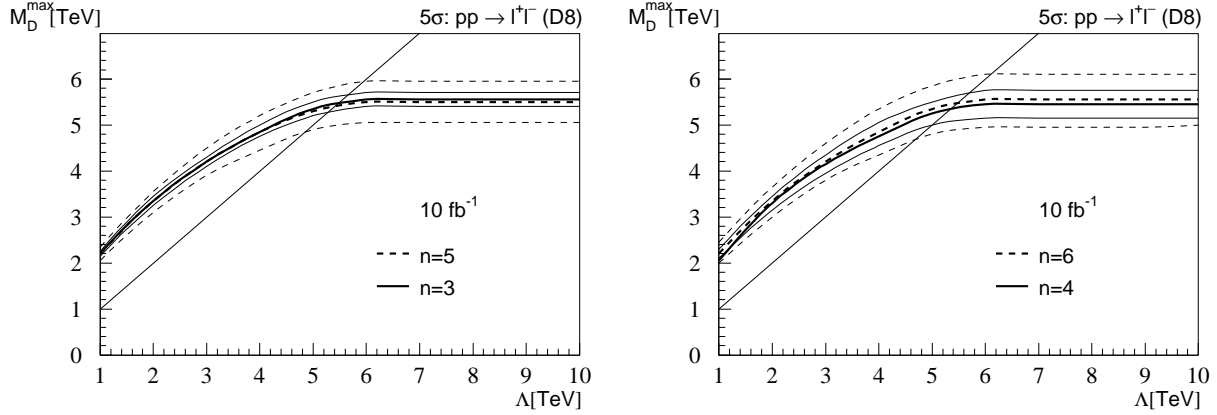


Fig. 1. The 5σ discovery contours in M_D at the LHC, shown as a function of a cutoff Λ on $\sqrt{s} = E_{\text{parton}}$ for an assumed integrated luminosity of 10 fb^{-1} . Thin lines show a $\pm 10\%$ variation of k_{trans} about M_D , the straight line is the diagonal $M_D^{\text{max}} = \Lambda$. The leveling-off at $M_D^{\text{max}} \approx \Lambda$ reflects the gravitational UV fixed point. To enhance the reach we require $m_{\ell\ell}^{\text{min}} = \min(M_D/3, 2 \text{ TeV})$.

Within asymptotically safe gravity the UV divergence in the m integration is regularized by the non-trivial anomalous dimension of the graviton. We implement this softening of gravity by evaluating the dressed propagator $1/(Z(k^2) p^2)$ at momentum scale $k^2 \approx p^2$ (p^2 denotes the relevant graviton momentum). Therefore, it scales like $p^{-2(1-\eta(p)/2)}$, which in the far UV becomes $(p^2)^{-D/2}$. For small s/M_D^2 , and because of the narrow crossover of the anomalous dimension, this amounts to the replacement

$$P(s, m) = \begin{cases} \frac{1}{s + m^2} & m < k_{\text{trans}} \\ \frac{k_{\text{trans}}^{n+2}}{(s + m^2)^{n/2+2}} & m > k_{\text{trans}} \end{cases} \quad (10)$$

The transition scale k_{trans} should be of the order of the fundamental Planck scale $k_{\text{trans}} \sim M_D$. The integration over m is finite, and for small s/M_D^2 it can be performed analytically, leading to $\mathcal{S}_{\text{FP}} = \mathcal{S}_A + \mathcal{S}_{\text{UV}}$. For large s/M_D^2 , we implement the leading asymptotic suppression of $\mathcal{S}(s)$ by matching with $\mathcal{S}_{\text{FP}}(s)$ at $s = k_{\text{trans}}^2$. Because of the steep decrease of the gluon density towards large \sqrt{s} the numerical impact of the details of this modelling can be expected to be small. For a more detailed evaluation, see [27].

4 LHC Signal

In Fig. 1 we display the discovery potential in M_D at the LHC. Taking into account the leading Z -production background we compute the minimal signal cross section $\sigma_{\text{tot}}(M_D)$ for which we can still observe a 5σ excess (see [26] for technical details). This minimal cross section translates into a reach M_D^{max} . The behavior of this prediction as an extension of the effective-field-theory method we check by introducing an artificial cutoff Λ on the partonic energy [26], setting $\mathcal{S}_{\text{FP}} = 0$ for $\sqrt{s} > \Lambda$. This cutoff is an unnecessary addition to our approach, which means that for sufficiently large

values, M_D^{max} like any observable has to become independent of it. This is nicely seen in Fig. 1. To estimate the uncertainties in our computation, we allow for a 10% variation in $k_{\text{trans}} \sim M_D$, leading to mild variations in Fig. 1 of a similar magnitude, slightly increasing with n .

In Fig. 2 we show the normalized \sqrt{s} or E_{parton} distributions for Drell-Yan production including all Standard-Model and KK graviton contributions for $n = 3$ and $M_D = 5 \text{ TeV}$ and 8 TeV . To show the entire range of \sqrt{s} , in contrast to Fig. 1 we do not apply any $m_{\ell\ell}$ cut. The solid curves represent our fixed-point analysis. The entire \sqrt{s} range contributes to the rate as long as there is a sizeable parton luminosity and as long as the large- s suppression of \mathcal{S} is not too strong. The dashed curves correspond to the cut-off approximation above $\sqrt{s} = \Lambda$, so there is no contribution above $\sqrt{s} = \Lambda$. The two sets of curves do not scale in a simple manner because Standard Model and KK amplitudes interfere. For small \sqrt{s} this interference term is significant, whereas for large \sqrt{s} there is hardly any Standard-Model contribution. Once we apply a cut of the kind $m_{\ell\ell} > M_D/3$ this background-interference contribution will become negligible.

In Tab. 1, we show the LHC production cross-section for the Drell-Yan process, including virtual gravitons, for $n = 3, 6$. Our fixed-point results \mathcal{S}_{FP} are given in (a) and (b): in (a), we simply retain the leading term in s/M_D^2 for all values of \sqrt{s} . In (b), we correct the high-energy behavior using the matched large- \sqrt{s} behavior above M_D . In (c) we introduce a double cut-off $\Lambda = M_D$ in m and \sqrt{s} into the naive KK effective theory [26]. For large enough $M_D \approx 5 - 8 \text{ TeV}$ we see that the LHC has little sensitivity to quantum-gravity effects in \sqrt{s} , and we find only small differences between (a) and (b). In that case, the difference between (b) and (c) is exclusively due to the KK integration. For small $M_D \approx 2 \text{ TeV}$, all three approaches lead to significant differences which originate from physics be-

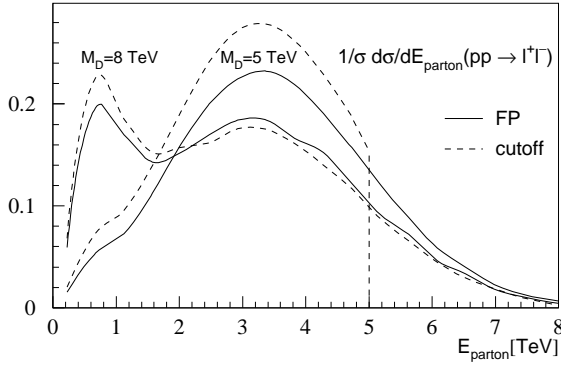


Fig. 2. Comparison of normalized distributions of the partonic energy E_{parton} for the dimension-8 operator correction to Drell-Yan production at the LHC ($n = 3$). Full line: present work, dashed line: approximation eq.(9) with $\Lambda = M_D$.

	$n = 3$			$n = 6$		
	2 TeV	5 TeV	8 TeV	2 TeV	5 TeV	8 TeV
<i>a</i>	2270	1.41	0.0317	2220	1.36	0.031
<i>b</i>	408	1.24	0.0317	398	1.21	0.031
<i>c</i>	173	0.72	0.0204	66	0.28	0.008

Table 1. Comparison of Drell-Yan production rates at the LHC after cuts for $M_D = 2, 5, 8$ TeV. See main text for the definitions of the scenarios (*a*), (*b*) and (*c*).

yond the fundamental Planck scale, which is omitted in the effective-field-theory approach (*c*).

5 Conclusions

We have laid out a framework to study quantum-gravitational effects at high energies within Wilson's renormalization group. This extends previous effective-field-theory computations towards momentum regimes at and above the fundamental Planck scale. Our approach is based on the dominant effects in asymptotically safe gravity. It can be extended to take vertex corrections into account [14], in ways similar to the systematics developed in other theories, *e.g.* infrared QCD [28]. For the physical observables studied here, we expect vertex corrections to be subleading because the relevant momentum integrals are dynamically suppressed above the Planck scale.

We employed our approach to gravitational Drell-Yan production in scenarios with large extra dimensions. The main new effects are dictated by the gravitational UV fixed point above the fundamental Planck scale. The renormalization-group improvement advocated here leads to finite cross-section and to theoretically well controlled experimental signatures at the LHC, already at low luminosities. The (model-dependent) UV contributions to the dimension-8 operator studied here may allow to distinguish different models for quantum gravity.

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